SS-3

A New Look at the 3D Condensed Node TLM Scattering

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Abstract

A systematic impulse numbering scheme and impulse splitting operation are developed. The new numbering scheme reveals the physics of the scattering procedure. The impulse splitting operation allows new scattering matrices of special nodes to be derived in a straightforward manner.

Introduction

Since P. B. Johns published his first paper on the symmetrical condensed node TLM scattering [1], researchers have been using the impulse numbering scheme given in that paper. The numbering scheme is not quite systematic. As a result the scattering matrix elements do not relate to each other in a systematic manner, and thus the scattering matrix fails to give special insight into the physics of the node. Russer has used a different numbering scheme in [2], thus perceiving the need for improvements in the numbering scheme, but he did not elaborate on it further. The numbering scheme presented in this paper is quite systematic, hence it provides insight into the scattering procedure as well as facilitates the derivation of new scattering matrices for condensed nodes with special properties.

Impulse Numbering Scheme

Our new numbering scheme is based on the right-hand rule used in vector calculus. We consider the numbering scheme for the node without stubs first and then generalize it to the fully loaded node.

The impulses travelling along the x-axis are incident from either the x- or x+ direction and their polarizations are parallel to either the y or z-axis, Figure 1. We name these impulses V_y^{x-}, V_z^{x-}, V_y^{x+} and V_z^{x+}; V_y^{x-} stands for the voltage impulse incident from the x- direction and having a polarization parallel to the y-axis, and so on. Similarly the impulses travelling along the y and z-axis are named V_z^{y-}, V_x^{y-}, V_z^{y+}, V_x^{y+}, V_x^{z-}, V_y^{z-}, V_x^{z+} and V_y^{z+}, respectively. These voltage names correspond to V_3, V_6, V_{11}, V_{10}, V_5, V_1, V_7, V_{12}, V_2, V_4, V_9 and V_8 in [1]. Table 1 gives the relationship between the new and old numbering schemes. The new reorganized scattering matrix S is:

$$S = \frac{1}{2}\begin{bmatrix}
 & & & & & 1 & & -1 & & 1 & & 1 \\
 & & & & 1 & & 1 & & 1 & & -1 & \\
 & & & & & -1 & & 1 & & 1 & & 1 \\
 & & & & 1 & & 1 & & -1 & & 1 & \\
 & 1 & & 1 & & & & & & 1 & & -1 \\
1 & & -1 & & & & & & 1 & & 1 & \\
 & 1 & & 1 & & & & & & -1 & & 1 \\
-1 & & 1 & & & & & & 1 & & 1 & \\
 & 1 & & -1 & & 1 & & 1 & & & & \\
1 & & 1 & & 1 & & -1 & & & & & \\
 & -1 & & 1 & & 1 & & 1 & & & & \\
1 & & 1 & & -1 & & 1 & & & & &
\end{bmatrix}$$

The above scattering matrix has four interesting properties which are the direct benefit of this new impulse numbering scheme:

(1) The 12×12 scattering matrix can be partitioned in to a 3×3 matrix system; each element in that matrix system is a 4×4 matrix:

$$S = S^{-1} = S^T = \frac{1}{2}\begin{bmatrix} & A & B \\ B & & A \\ A & B & \end{bmatrix}$$

where A and B are:

$$A = \begin{bmatrix} & 1 & & -1 \\ 1 & & 1 & \\ & -1 & & 1 \\ 1 & & 1 & \end{bmatrix} \quad \text{and} \quad B = A^T = \begin{bmatrix} & 1 & & 1 \\ 1 & & -1 & \\ & 1 & & 1 \\ -1 & & 1 & \end{bmatrix}$$

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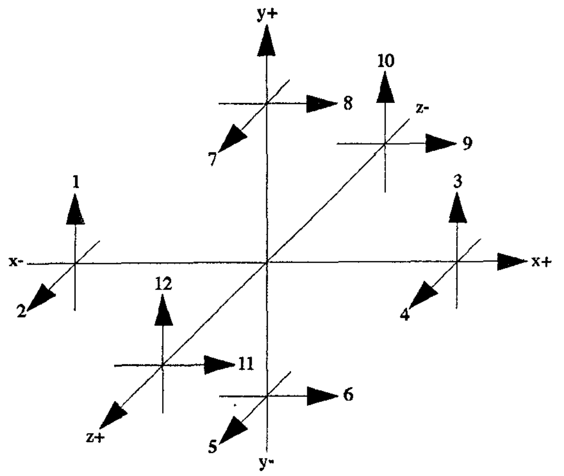

Figure 1: A new condensed node impulse numbering scheme.

Impulse Names	New Scheme	P.B. Johns' Scheme
V_y^{x-}	V_1	V_3
V_z^{x-}	V_2	V_6
V_y^{x+}	V_3	V_{11}
V_z^{x+}	V_4	V_{10}
V_z^{y-}	V_5	V_5
V_x^{y-}	V_6	V_1
V_z^{y+}	V_7	V_7
V_x^{y+}	V_8	V_{12}
V_x^{z-}	V_9	V_2
V_y^{z-}	V_{10}	V_4
V_x^{z+}	V_{11}	V_9
V_y^{z+}	V_{12}	V_8

Table 1: Relationship between the new and old impulse numbering scheme for the condensed node. V_y^{x-} stands for the voltage impulse incident from the x- direction and having a polarization parallel to the y-axis, and so on.

(2) The matrices in the matrix system are A and B, and $B = A^T$. This is because the node is symmetrical, and the right-hand rule is used.

(3) The 4×4 diagonal matrices of this 3×3 matrix system are zero matrices. This is so because impulses travelling along one axis are scattered into the other two axial directions only.

(4) The elements of the second and third row of the matrix system are the cyclic permutation of the elements of the first row; this is a direct benefit of using the right-hand rule.

Impulse Splitting Procedure

The above scattering matrix can be expanded by logically splitting each impulse of the node into two parts, say a and b as shown in Figure 2. By replacing the elements in A and B with their equivalent 2×2 zero and identity matrices, the 4×4 matrices of the above 3×3 matrix system become 8×8 matrices.

This expanded scattering matrix can be used to derive new scattering matrices for condensed nodes with special properties. For example, it allows us to find, almost by inspection, the scattering matrix of a condensed node with a perfect conducting metallic plate of zero thickness in its xz-plane, Figure 3, [3].

Special Scattering Matrix

If there is a perfectly conducting metallic plate in the xz-plane of a condensed node (Figure-3), V_2, V_4, V_9 and V_{11} must be zero. To make sure that these voltages are zero, all the entries in the scattering matrix that will cause energy to be scattered into them as well as all the entries that will cause energy to be scattered out of them must be set to zero. The entries in the scattering matrix that will cause top-down interaction must also be set to zero. Finally, because of the presence of the conducting plate, energy which would normally be scattered to V_{8b} from V_{1b} must now go to V_{6b}, and so on. The resulting scattering matrix S is:

$$S = \frac{1}{2}\begin{bmatrix} & A & B \\ C & & D \\ B^T & E & \end{bmatrix}, \text{ where } A, B, C, D \text{ and } E \text{ are: } A = \begin{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & 1 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} -1 & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & -1 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} 1 & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \end{bmatrix}, B = \begin{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \end{bmatrix},$$

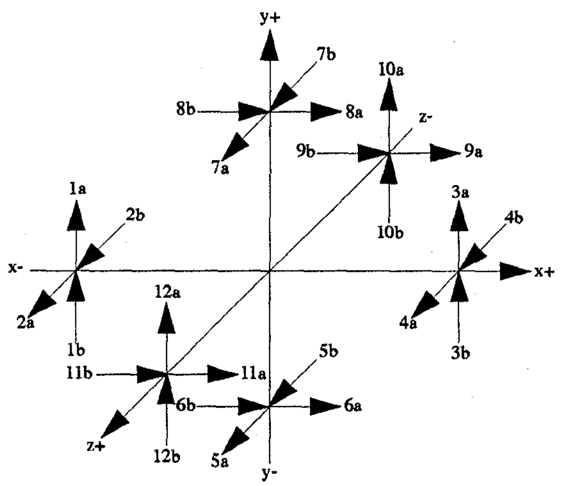

Figure 2: Logically splitting impulses into two parts, those closer to the voltage labels are part *a* and the tailing ones are part *b*. The total voltage of each individual impulse is the sum of part *a* and part *b*; $V_1 = V_{1a} + V_{1b}$, and so on.

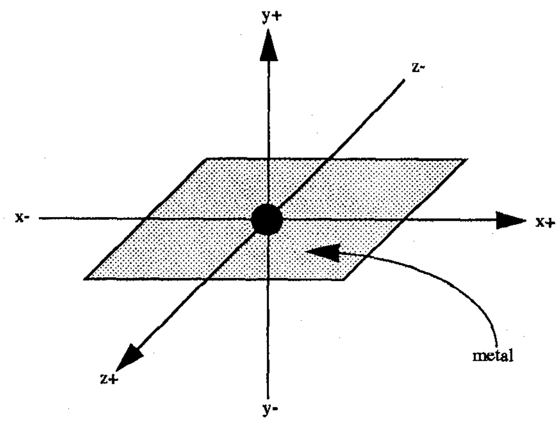

Figure 3: Nielsen's half-node for S_{hy}

$$C = \begin{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & 2 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & -2 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \\ \begin{bmatrix} -2 & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} 2 & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \end{bmatrix}, \quad D = \begin{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & 2 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & -2 \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} -2 & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} 2 & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \end{bmatrix}, \quad E = \begin{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & 1 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & -1 \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \\ \begin{bmatrix} & \\ & -1 \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} & \begin{bmatrix} 1 & \\ & \end{bmatrix} & \begin{bmatrix} & \\ & \end{bmatrix} \end{bmatrix}$$

This scattering matrix is equivalent to the S_{hy} matrix in [3], but is derived with considerably less effort and is easily permuted to find S_{hx} and S_{hz}.

Lossy Node Numbering Scheme

We have generalized this new impulse numbering scheme for the fully loaded symmetrical condensed lossy node [4], and its scattering matrix is:

$$S = \frac{1}{2}\begin{bmatrix} A_x & B_x & C_x \\ C_y & A_y & B_y \\ B_z & C_z & A_z \end{bmatrix}$$

; where $A_x, B_x, C_x, A_y, B_y, C_y, A_z, B_z$ and C_z are 7×7 matrices with the elements in the seventh column being all zeros.

$$A_x = \begin{bmatrix} a_{yz} & & c_{yz} & & & & \\ & a_{zy} & & c_{zy} & & & \\ c_{yz} & & a_{yz} & & & & \\ & c_{zy} & & a_{zy} & & & \\ & & & & h_x & & \\ & & & & & j_x & \\ & & & & p_x & & \end{bmatrix}, \quad B_x = \begin{bmatrix} & d_z & & -d_z & g_y & & \\ b_z & & b_z & & & i_y & \\ & -d_z & & d_z & g_y & & \\ b_z & & b_z & & & -i_y & \\ & e_x & & e_x & & & \\ -f_x & & f_x & & & & \\ & k_x & & k_x & & & \end{bmatrix} \quad \text{and} \quad C_x = \begin{bmatrix} & b_y & & b_y & & -i_z & \\ d_y & & -d_y & & g_z & & \\ & b_y & & b_y & & i_z & \\ -d_y & & d_y & & g_z & & \\ e_x & & e_x & & & & \\ & f_x & & -f_x & & & \\ k_x & & k_x & & & & \end{bmatrix}.$$

Impulse Names	New Scheme	Scheme in [4]
V_y^{x-}	V_1	V_3
V_z^{x-}	V_2	V_6
V_y^{x+}	V_3	V_{11}
V_z^{x+}	V_4	V_{10}
V_{ex}	V_5	V_{13}
V_{hx}	V_6	V_{16}
V_{gx}	V_7	V_{19}
V_z^{y-}	V_8	V_5
V_x^{y-}	V_9	V_1
V_z^{y+}	V_{10}	V_7
V_x^{y+}	V_{11}	V_{12}
V_{ey}	V_{12}	V_{14}
V_{hy}	V_{13}	V_{17}
V_{gy}	V_{14}	V_{20}
V_x^{z-}	V_{15}	V_2
V_y^{z-}	V_{16}	V_4
V_x^{z+}	V_{17}	V_9
V_y^{z+}	V_{18}	V_8
V_{ez}	V_{19}	V_{15}
V_{hz}	V_{20}	V_{18}
V_{gz}	V_{21}	V_{21}

Table 2: Relationship between the new and old impulse numbering scheme for the fully loaded condensed lossy node. V_{ex} and V_{hx} stand for voltage impulses coupling with the x-component of the electric and magnetic fields. V_{gx} account for the dielectric loss associated with the x-component of the electric field, and so on.

A_y, B_y, C_y, A_z, B_z and C_z are cyclic permutations of A_x, B_x and C_x. As an example:

$$A_y = \begin{bmatrix} a_{zx} & & c_{zx} & & & & \\ & a_{xz} & & c_{xz} & & & \\ c_{zx} & & a_{zx} & & & & \\ & c_{xz} & & a_{xz} & & & \\ & & & & h_y & & \\ & & & & & j_y & \\ & & & & p_y & & \end{bmatrix}$$

The coefficients in the above A, B and C matrices are:

$$a_{mn} = -\frac{G_m+Y_m}{G_m+Y_m+4}+\frac{Z_n}{Z_n+4}$$

$$b_m = e_m = k_m = \frac{4}{G_m+Y_m+4}$$

$$c_{mn} = -\frac{G_m+Y_m}{G_m+Y_m+4}-\frac{Z_n}{Z_n+4}$$

$$d_m = i_m = \frac{4}{4+Z_m}$$

$$h_m = -2\frac{G_m-Y_m+4}{G_m+Y_m+4}$$

$$j_m = 2\frac{4-Z_m}{4+Z_m}$$

$$f_m = Z_m d_m$$

$$g_m = p_m = Y_m b_m$$

Table 2 shows the relationship between the old and new numbering schemes for the fully loaded condensed lossy node.

Conclusion

We have used this new impulse numbering scheme and the impulse splitting operation to derive a number of scattering matrices for some special condensed nodes. These special nodes can be used to model the sizes and positions of metallic boundaries with good accuracy without using an unnecessarily refined mesh. The impulse splitting operation can be generalized to model some boundary properties, such as sharp edges and corners, finite conductivity, skin effect and implementation of discrete devices into a TLM network, which can not be modelled easily and accurately by placing the boundary half-way between nodes.

References

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